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# Learning Constraint Networks over Unknown Constraint Languages

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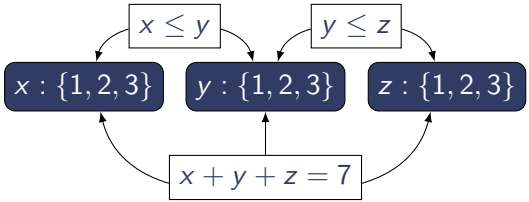
# Background

A **constraint network**:

- a set of **variables** over a finite **domain**
- a set of **constraints**, i.e. relations between variables that must be satisfied in any **solution**



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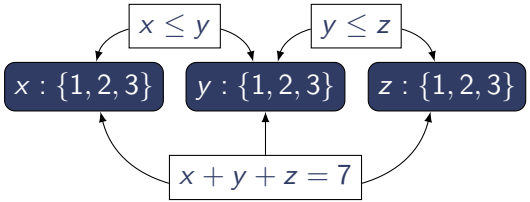


A **constraint language** is a set of relations over a domain.

- Language of a constraint network : set of all relations that appear in its constraints

# Constraint Programming & Constraint Acquisition

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**Challenge** | Designing a constraint network representing a given problem can be difficult.

- To overcome this, **constraint acquisition** learns a constraint network automatically.

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ConAcq.1 (Bessiere et al., 2006, 2017), ModelSeeker (Beldiceanu and Simonis, 2012), BayesAcq (Prestwich et al., 2021), Count-CP (Kumar et al., 2022)

**Problem** | All current approaches require some knowledge of the constraint language of the output network.

# Our contribution

We develop a constraint acquisition method that
**constructs a constraint language as part of the learning process**.

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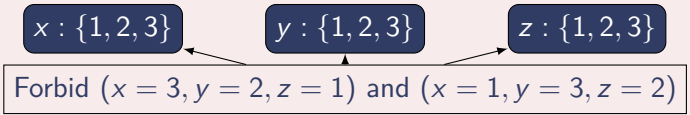

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**Problem** | Language-Free Acq

**Instance:** Set of examples, labelled as solutions and non-solutions, two integers $k$ and $r$.

**Question:** Is there a constraint network **over a language of size at most** $k$ **and arity at most** $r$ that correctly classifies the examples?

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Language-Free Acq is NP-complete even for $(k, r) = (1, 1)$.

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- **Tie-breaking:** lower arity, more constraints, tighter constraints
- **Construct and solve a model for each** $(k, r)$**:**
  - Convert to an instance Weighted Partial Max-Sat
  - Compute the optimal network or prove that none exists
  - Output the first constraint network found

# Experimental Results

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- Learn an **equivalent network** on another language:
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  - Golomb Ruler (with 10 variables) [3200 examples]
- Neither learn the target language nor an equivalent network:
  - 8-Queens (coordinates model)

# Recap and Future Work

We proposed a novel constraint acquisition method that
**does not require any knowledge on the constraint language of the target network**.


Experiments show promising results, but has limitations. Could be addressed by using **more sophisticated notions of simplicity** and **detecting topological information**.

Thank you for your time and attention.

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Experiments performed with the MESO@LR-Platform at University of Montpellier

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