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Learning Compact Representations of Constraint Networks

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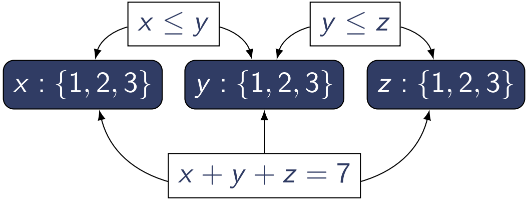
Background

A **constraint network**:

- a set of **variables** over a finite **domain**
- a set of **constraints**, i.e. relations between variables that must be satisfied in any **solution**



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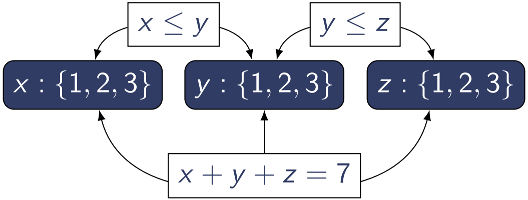

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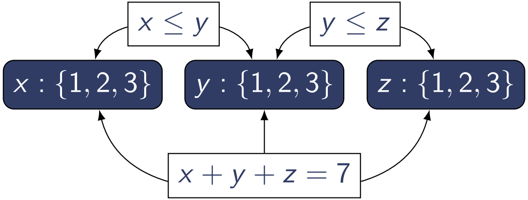

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Challenge: The modeling process is a significant bottleneck

The Solution: Constraint Acquisition

Passive Constraint Acquisition

Instance: Set of examples (assignments labelled as solutions and non-solutions).

Goal: Find a constraint network consistent with the examples.

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CONACQ.1 (Bessiere et al., 2006, 2017), MODELSEEKER (Beldiceanu and Simonis, 2012), BAYESACQ (Prestwich et al., 2021), COUNT-CP (Kumar et al., 2022), LANGUAGE-FREE ACQ (Bessiere et al., 2023), etc.

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Which network will generalize best to new, unseen data?

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Our claim

Learning compact representations of constraint networks is the key to better generalization.

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Our Contribution

1. A novel, compact representation for structured networks, which we call *template*.
2. A new acquisition framework, TACQ, that learns these templates directly from examples.

Compact Representation

Example with the Sudoku

(a) Attribute 1 (Rows) (b) Attribute 2 (Columns) (c) Attribute 3 (Squares 3×3)

- For each variable (cell), we specify three *attributes* (ϕ_{row}, ϕ_{col}, and ϕ_{square}).
- We produce all the constraints of Sudoku using *rules* depending on these attributes.

What is a Template?

1. **Attributes:** Functions that assign numerical features to variables.

2. **Rules:** Mechanisms for producing many constraints based on attributes. It consists of:
   - A **Relation** for the constraint (e.g., $\neq$),
   - A **Selector** that specifies which attributes to check,
   - A **Trigger** function that determines if the constraint should be produced based on the selected attributes.

Template Example: Sudoku

A Concise Representation for Sudoku

Variables:

- 81 variables : $x_{i,j}$

Attributes:

- $\phi_{\text{row}}(x_{i,j}) = i$
- $\phi_{\text{col}}(x_{i,j}) = j$
- $\phi_{\text{square}}(x_{i,j}) = \left\lfloor \frac{i}{3} \right\rfloor \times 3 + \left\lfloor \frac{j}{3} \right\rfloor$

Rules :

1. **Row Rule:**
   Apply '$\neq$' to (x_u, x_v) *if* $\phi_{\text{row}}(x_u) = \phi_{\text{row}}(x_v)$.
2. **Column Rule:**
   Apply '$\neq$' to (x_u, x_v) *if* $\phi_{\text{col}}(x_u) = \phi_{\text{col}}(x_v)$.
3. **Square Rule:**
   Apply '$\neq$' to (x_u, x_v) *if* $\phi_{\text{square}}(x_u) = \phi_{\text{square}}(x_v)$.

Learning Algorithm

The TACQ Learning Framework

A Two-Step Process:

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1. **Learn Initial Network:** Use a baseline method to learn an initial network N that is consistent with the training examples.

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1. **Learn Initial Network:** Use a baseline method to learn an initial network N that is consistent with the training examples.
2. **Refine into a Template:** Learn a compact template T whose interpretation is a large subset of N.

The Template Learning Algorithm

Algorithm Sketch

Input: A training set E and an initial network N consistent with E.

Output: A template T consistent with E.

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1. Start with an empty template T.
2. **While** the template T is not consistent with the training set E:
   1. **Guess new attribute.**
   2. **Greedily add rules that produce many new constraints of N.**

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2. **While** the template T is not consistent with the training set E:
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A key contribution is the heuristic which determines the width of new attributes.

Experimental Evaluation

Higher Accuracy / Fewer Examples

Problem	**Examples for 100% accuracy**		**Reduction**
	LFA	LFA+TACQ	
Sudoku	120	**80**	33%
Jigsaw [*3 instances*]	1490	**1130**	24%
Nurse Rostering [*3 instances*]	720	**590**	18%
Exam Timetabling [*3 instances*]	2536	**919**	64%
Schur's Lemma	560	560	0%
Subgraph Isomorphism	640	640	0%
Golomb Ruler (10 variables)	2100	2100	0%

Table: Number of examples to reach 100% accuracy.

Results: Learning Interpretable Attributes

TACQ learns attributes corresponding to meaningful features.

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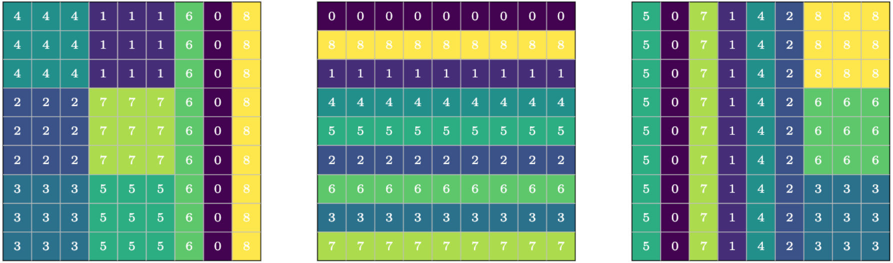

(a) Attribute 1 (ϕ_1) (b) Attribute 2 (ϕ_2) (c) Attribute 3 (ϕ_3)

Figure: Illustration of the three attributes learned by TAcq for Sudoku.

Conclusion

Key Takeaways

- Template is a representation of constraint networks in a compact and structured form,
- TACQ is a framework that learns Templates from examples,
- TACQ significantly **reduces the number of examples** needed to learn an accurate model for structured problems,
- The learned templates are often **interpretable**.

Future Work: Investigate learning parameterized models.

Thank you for your time and attention.

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