Réduction de la dépendance aux connaissances préalables en acquisition de contraintes

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CP involves two main steps:

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CP is widely used in scheduling, logistics, timetabling, etc.

Constraint Programming Workflow

Planning a Work Schedule (Nurse Rostering Problem)

Imagine you need to assign shifts to 15 nurses over a week with 3 shifts per day while respecting the following rules:

- Each shift must be covered by 3 nurses.
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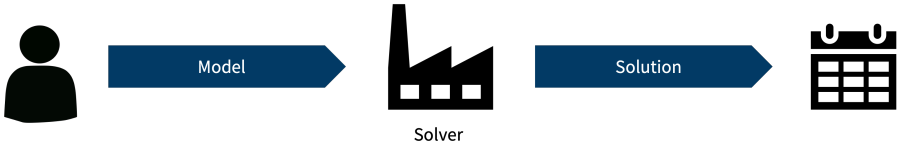

Variables and Domains

Definition | Variable

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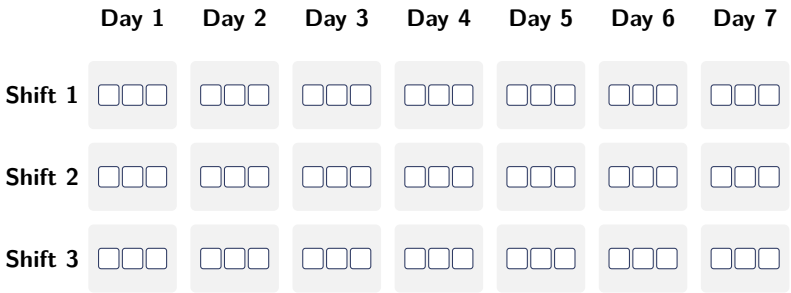
An **assignment** gives a value from the domain to each variable.

Example

In the nurse rostering problem:

- Variables X: $x_{i,j,k}$ is the nurse assigned to slot i of shift j on day k.
- Domain: $\{1, 2, \ldots, 15\}$ (available nurses)



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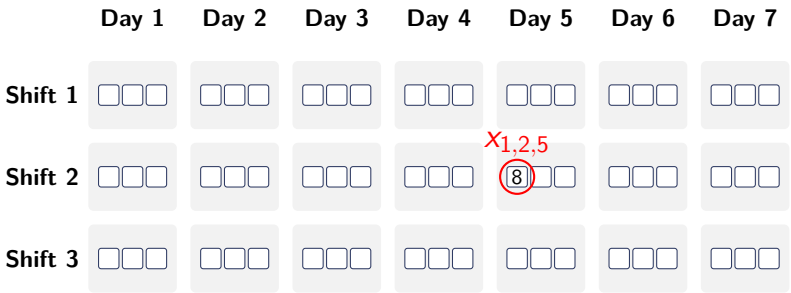

Constraint

Definition | Constraint

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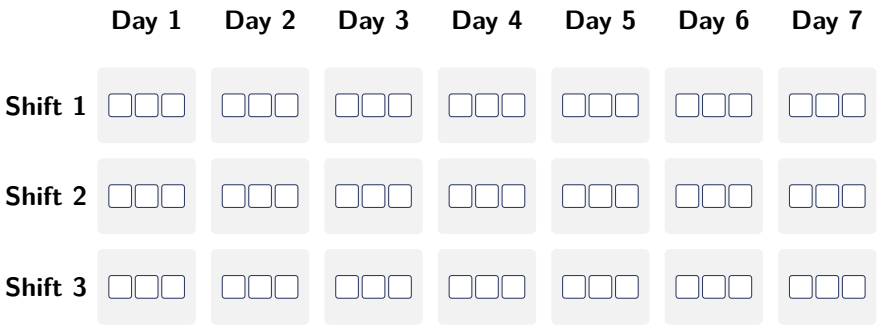
Example

- $S = (x_{1,2,5}, x_{2,2,5})$
- $R = \{(a, b) \mid a \neq b\}$

The Constraints of the Nurse Rostering Problem



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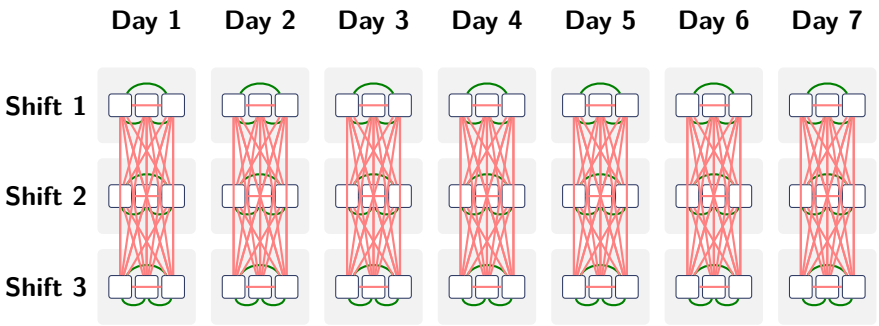

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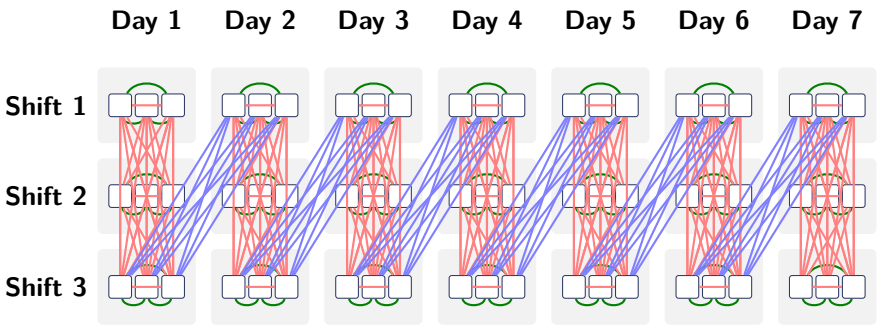

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A **constraint network** is a triplet $N = (X, D, C)$ where:

- X is a set of **variables** $\{x_1, \ldots, x_n\}$;
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Definition | Constraint Language

A **constraint language** Γ is a set of relations over a domain.

- A network N is over Γ if all constraints of N use relations from Γ.

Define the problem as a constraint network

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Challenge | Designing a constraint network representing a given problem can be difficult.

Examples and Consistency

Definition | Example

An example over a set of variables X with domain D is composed of:

- an assignment to X;
- a classification label as a *positive example* or as a *negative example*.

Definition | Consistency

A constraint network N is consistent with a set of examples E if:

- All positive examples in E are solutions of N.
- All negative examples in E are non-solutions of N.

Illustration of Passive Constraint Acquisition

Formal Definition

Definition | Passive Constraint Acquisition Task

Input:
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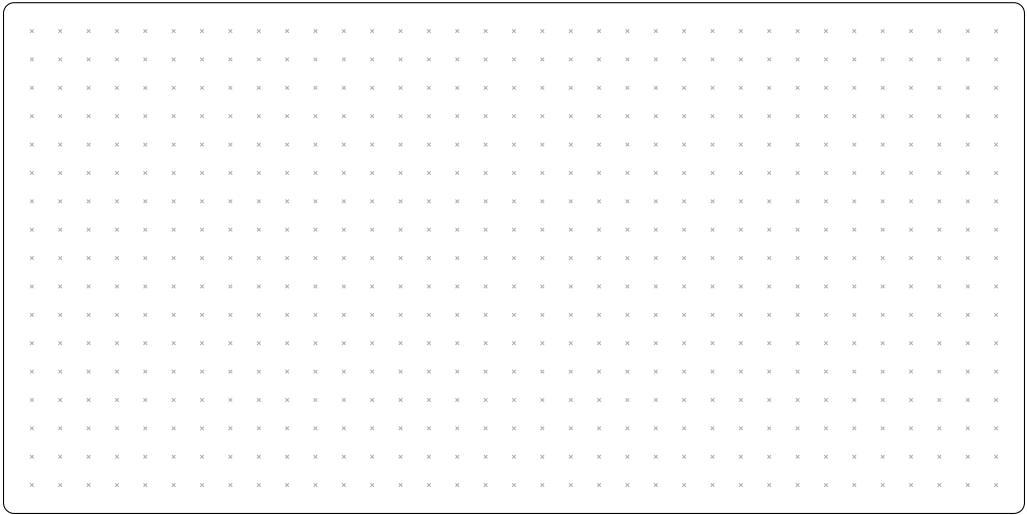
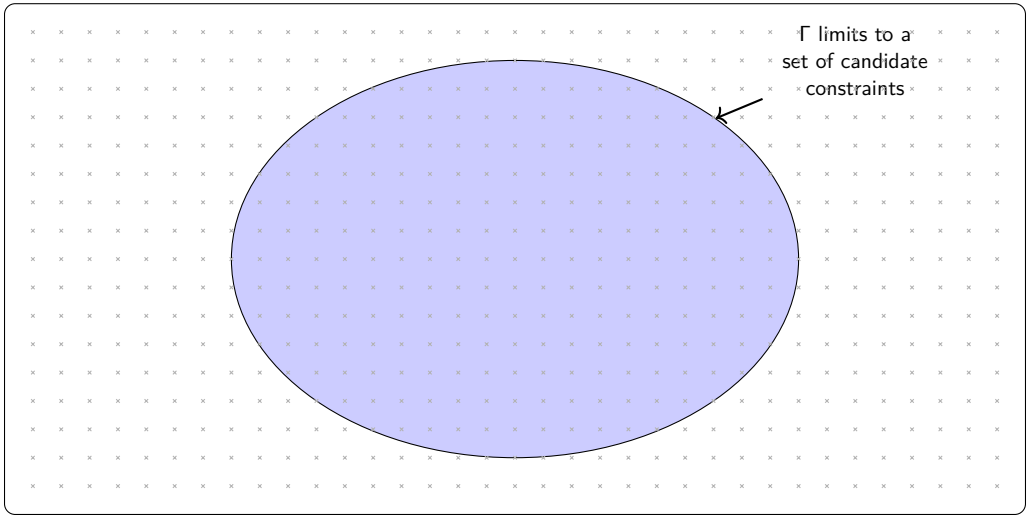
ConAcq.1 [Bessiere et al., 2005, 2017], ModelSeeker [Beldiceanu and Simonis, 2012], BayesAcq [Prestwich et al., 2021], Count-CP [Kumar et al., 2022]

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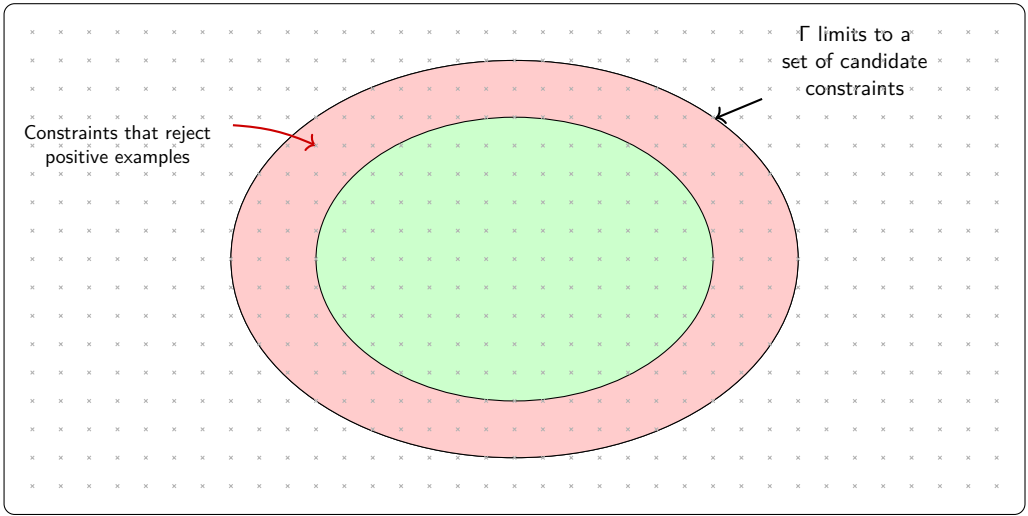
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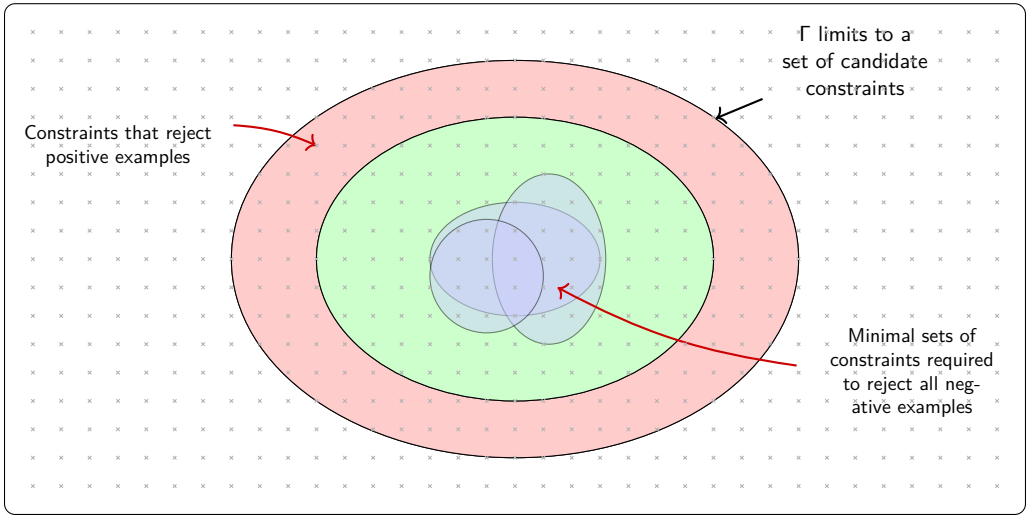
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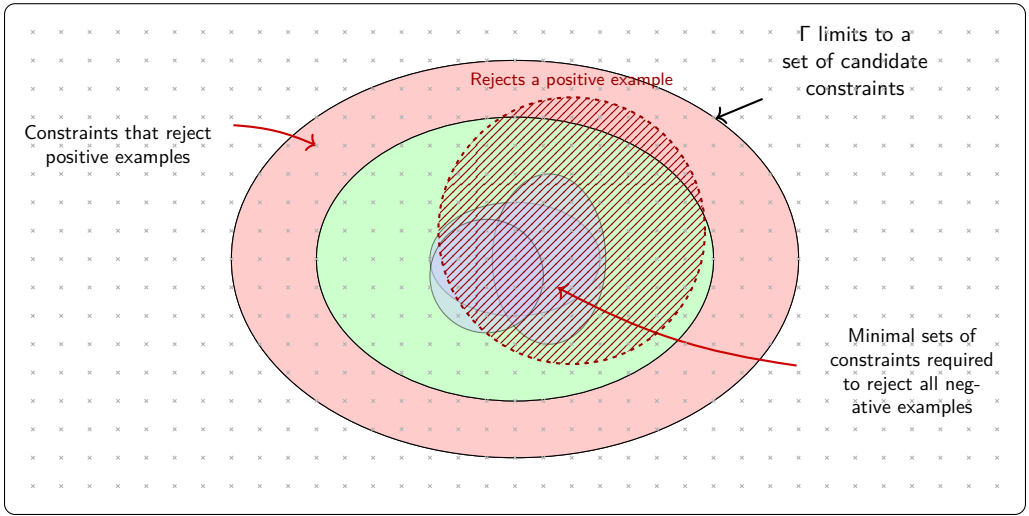
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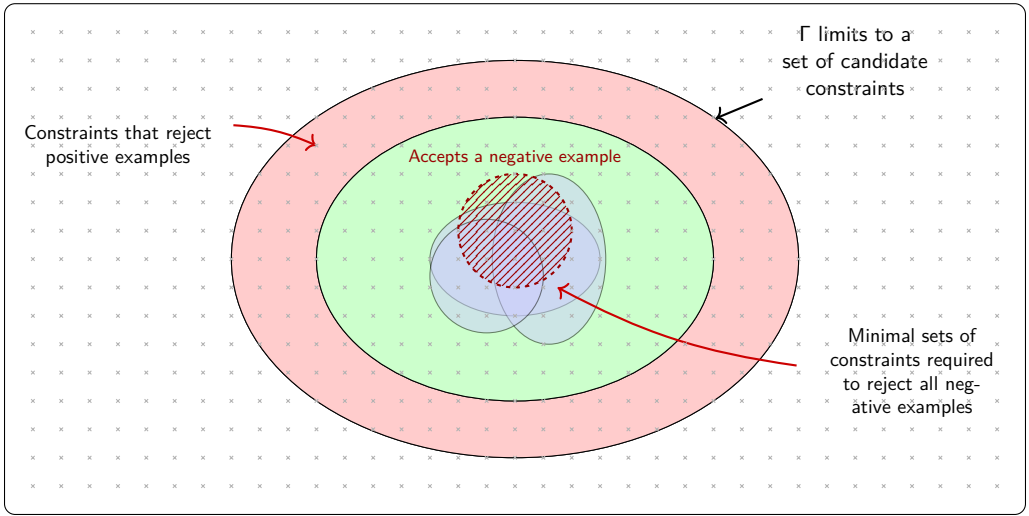
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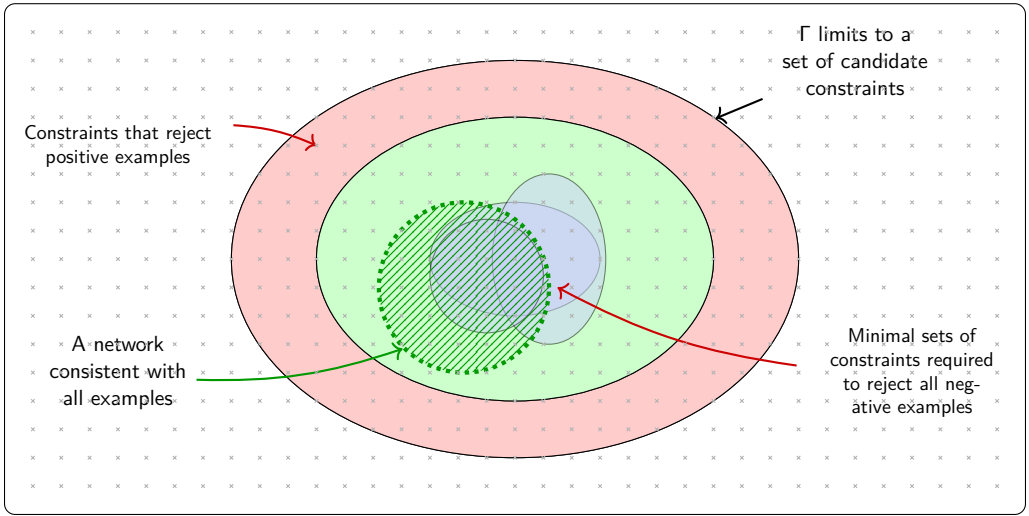
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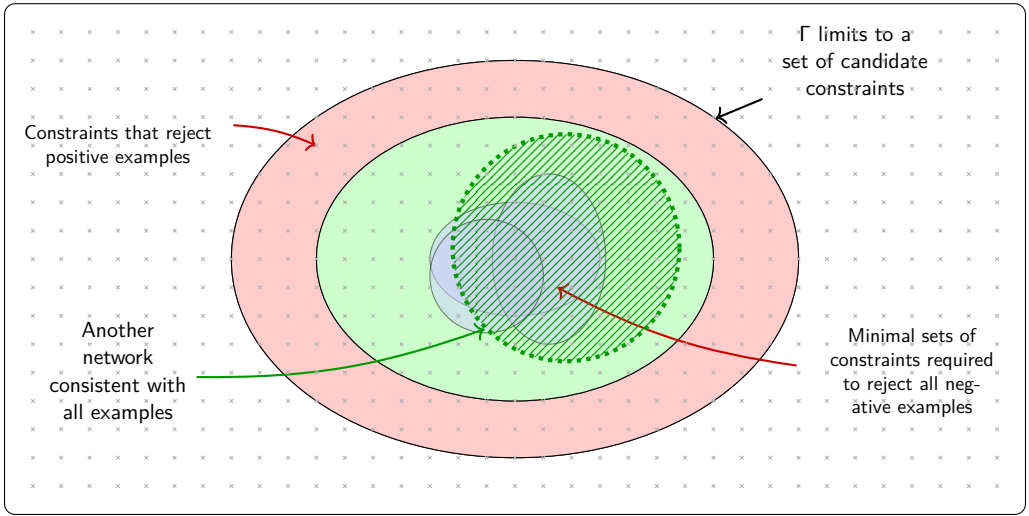
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Problem | Reliance on prior knowledge prevents automated modeling.

Motivation

All current approaches require some knowledge of the constraint language of the output network.

Our contribution

We develop a constraint acquisition method that
constructs a constraint language as part of the learning process.

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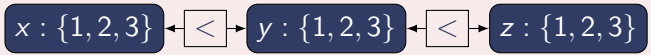

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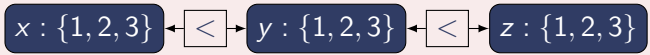

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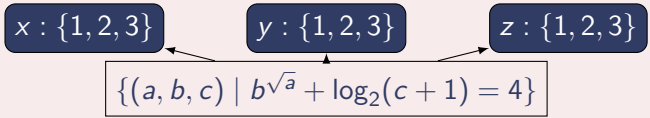

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Instance: Set of examples E; two integers k and r.

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This problem is NP-complete even for $(k, r) = (1, 1)$.

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Construct and solve a model for each (k, r) by increasing order.

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$$\text{Try } (k = 1, r = 1) \xrightarrow{\textit{fail}} (k = 2, r = 1) \xrightarrow{\textit{fail}} \cdots \xrightarrow{\textit{success}} \text{Output Network}$$

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- **Accuracy:** Count examples needed for **100% accuracy** (independent test set).
- **Qualitative:** Check if the learned network is:
  - the **target network**,
  - an **equivalent network**,
  - over the **target language**.

Summary of results over various benchmarks

Benchmark	Average number of examples for 100% accuracy	Language	Network
Sudoku	200	✓	✓
Jigsaw [*3 instances*]	900	✓	●
Nurse Rostering [*3 instances*]	467	✓	✓
Exam Timetabling [*3 instances*]	867	✓	✓
Schur's Lemma	600	✓	✓
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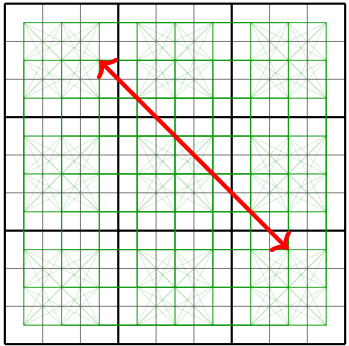
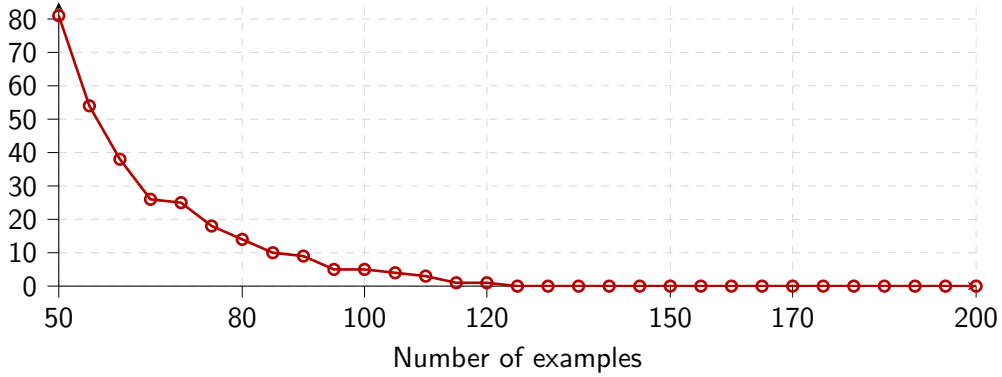
Residual Constraints

Residual constraints are those that are not in the target network but are consistent with all provided examples.

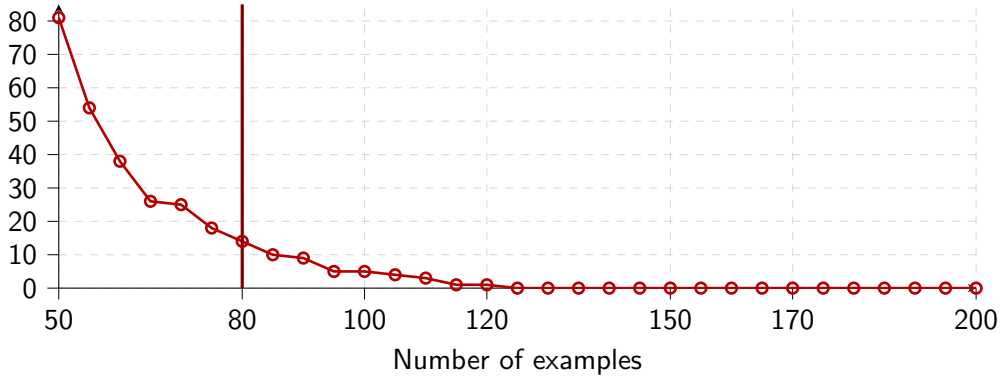


Residual Constraints



Number of residual constraints learned
as a function of the number of examples (Sudoku)

Residual Constraints



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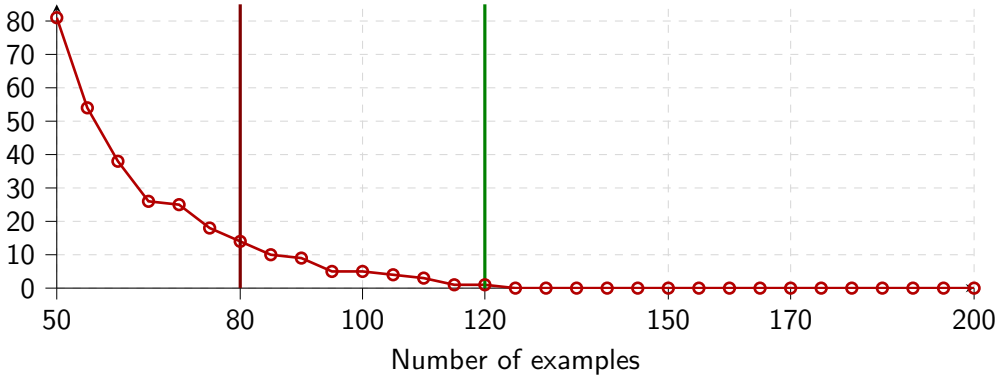

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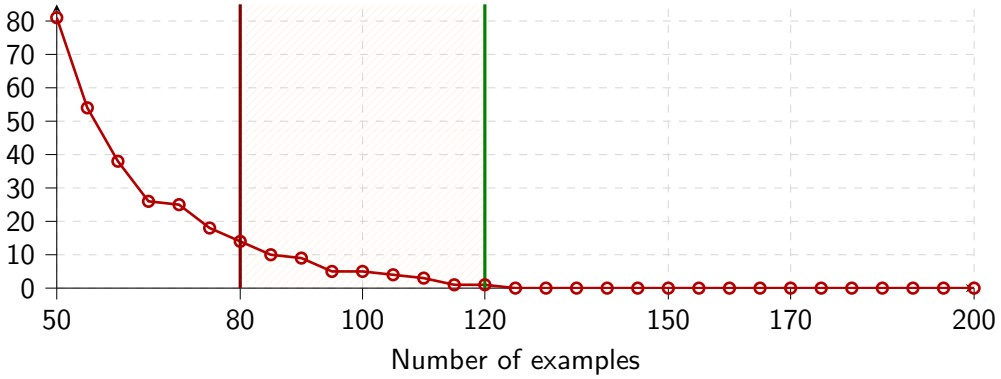

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Limitation: Lack of Structure

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- LFA does not try to capture the structure of the problem;
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Goal

Instead of learning constraints using a flat list of scopes:

$$x_{1,1} \neq x_{1,2},\ x_{1,2} \neq x_{1,3},\ \ldots$$

we want to learn rules such as:

"All variables in the same row must be different."

Learning Compact Representations

Our claim

Learning compact representations of constraint networks is the key to better generalization.

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Learning compact representations of constraint networks is the key to better generalization.

To this end, we propose:

1. A novel, compact representation for structured networks, which we call *template*.
2. A new acquisition framework, TACQ, that learns these templates directly from examples.

What is a Template?

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1. **Attributes:** Functions $\{\phi_1, \phi_2, \ldots\}$ that assign numerical features to variables.

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2. **Rules:** Mechanisms for producing many constraints based on attributes.
   - A relation (e.g., $\neq$),
   - A condition on some attributes of the variables (e.g., same attribute value).

   A rule produces a constraint for each scope of variables satisfying the condition.

Example | Template for the Sudoku

Variables: 81 variables $\{x_{i,j} \mid i, j \in [0..8]\}$

Attributes: $\phi_{\text{row}}(x_{i,j}) = i$; $\phi_{\text{col}}(x_{i,j}) = j$; $\phi_{\text{square}}(x_{i,j}) = \left\lfloor \frac{i}{3} \right\rfloor \times 3 + \left\lfloor \frac{j}{3} \right\rfloor$

ϕ_{row} ϕ_{col} ϕ_{square}

Rules:

1. Apply $\neq$ to (x, y) *if* $\phi_{\text{row}}(x) = \phi_{\text{row}}(y)$.
2. Apply $\neq$ to (x, y) *if* $\phi_{\text{col}}(x) = \phi_{\text{col}}(y)$.
3. Apply $\neq$ to (x, y) *if* $\phi_{\text{square}}(x) = \phi_{\text{square}}(y)$.

The TAcq Learning Framework

A two-step process:

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The TAcq Learning Framework

A two-step process:

1. **Learn an initial network:** Use a baseline method to learn an initial network N
2. **Refine this network into a template:** Learn a template that produces a large subset of the constraints of N.

The Template Learning Algorithm

Algorithm sketch

Input: A set of examples E and an initial network N consistent with E.

Output: A template T consistent with E.

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   2. **Greedily add rules that produce many new constraints of** N

Attribute Width and Generalization

The number of distinct values an attribute takes (its **width**) affects the maximum number of constraints produced by a rule based on that attribute (the **coverage**).

Width too small

Underfitting

Few constraints are produced.
We fail to capture the problem.

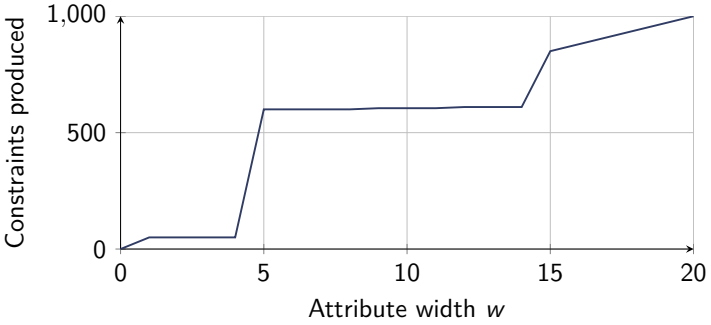
Width too large

Overfitting

Too many constraints are produced.
We capture the residual constraints.

The Maximum Cover Above Expectation (MCAE) heuristic

The MCAE heuristic looks for the attribute with a trade-off between width and coverage.



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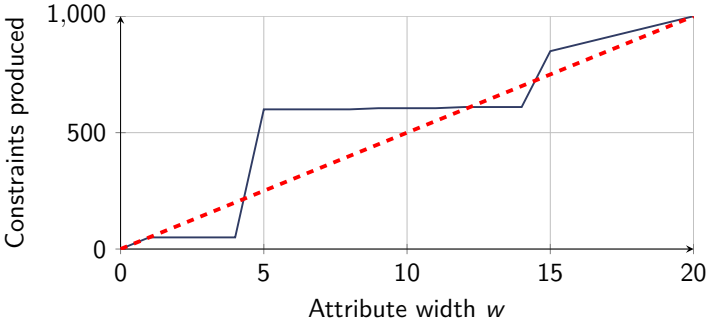

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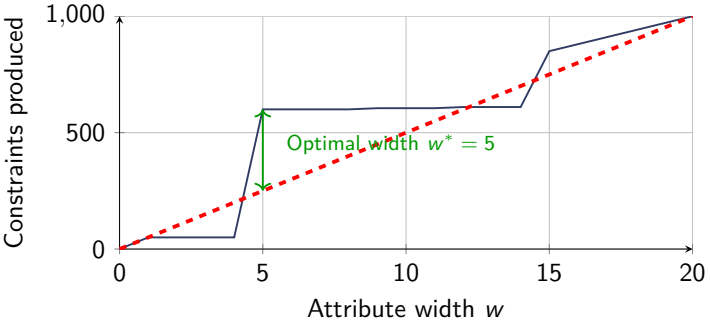

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Protocol

- We generate an ordered sequence of examples,
- Both methods learn from the same sequence,
- We record the number of examples needed to reach 100% accuracy.

Experimental Evaluation

Problem	**Examples for 100% accuracy**		**Reduction**
	LFA	LFA+TACQ	
Sudoku	120	**80**	33%
Jigsaw [*3 instances*]	497	**377**	24%
Nurse Rostering [*3 instances*]	240	**197**	18%
Exam Timetabling [*3 instances*]	845	**306**	64%
Schur's Lemma	560	560	0%
Subgraph Isomorphism	640	640	0%
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Results: Learning Interpretable Attributes

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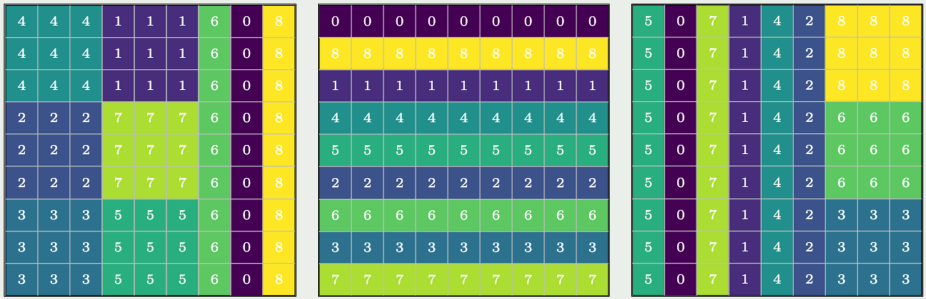

Figure: Illustration of the three attributes learned by TACQ for Sudoku.

1 | Refined simplicity of relations

Path: Define simplicity with a construction cost.

- Define a basis of primitives (e.g., $\{=, <, >, \ldots\}$);
- Favor relations constructible from these primitives with few operations.

2 | Better interpretability of attributes

Path: Bias towards interpretable attributes.

- Leverage statistical analysis of existing CP models (e.g., CSPLib);
- Guide learning to prefer this attribute distribution.

3 | Robustness to noise

Limitation: LFA/TACQ treat examples as ground truth.

- In some scenarios labels may be incorrect;
- **Path**: Relax the Max-SAT model.

Minimize (*Model Complexity* + *Examples Classification Error*)

4 | New class of oracles

Challenge: Use **black-box models** (e.g., CNNs) as oracles instead of humans.

Path: Leverage explanation methods (e.g., Grad-CAM for CNNs) to guide acquisition.

Conclusion

Summary

Problem: Reliance on prior knowledge prevents automated modeling.

- LFA: Enables learning without a language.
- TAcq: Recovers structure for better generalization.

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- LFA: Enables learning without a language.
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LFA and TAcq are publicly available:

LFA

Published at IJCAI-2023

Source code: language-free-acq

```
pip install languageFreeAcq
```

TAcq

Published at ECAI-2025

Source code: TAcq

```
pip install tacq
```

Thank you for your time and attention.

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